Critical Scale Invariance in a Healthy Human Heart Rate

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We demonstrate the robust scale-invariance in the probability density function (PDF) of detrended healthy human heart rate increments, which is preserved not only in a quiescent condition, but also in a dynamic state where the mean level of the heart rate is dramatically changing. This scale-independent and fractal structure is markedly different from the scale-dependent PDF evolution observed in a turbulentlike, cascade heart rate model. These results strongly support the view that a healthy human heart rate is controlled to converge continually to a critical state.

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A healthy human heart rate belongs to a special class of complex signals, showing long-range temporal correlations [1,2], non-Gaussianity of the increment's probability density function (PDF) [1] and multifractal scaling properties [3,4] and has served as the "benchmark" of choice for studies of biological complexity. Two alternative mechanisms, both characterized by these three features, have been proposed for this heart rate complexity: (i) a random (multiplicative) cascade process based on the resemblance of the behavior of the structure function [5] of heart rate increments to that of spatial velocity differences in hydrodynamic turbulence [6] and (ii) critical statelike dynamics [7] based on the resemblance of the scale-invariant properties in heart rate to those of many systems operating near the critical point of their phase space. To date the exact mechanism for the complex heart rate dynamics is unknown. However, as it reflects the dynamics of the autonomic nervous system's control of heart rate [1,4] and thus provides potential predictors for the mortality of cardiac patients [8–10], elucidating this mechanism is considered important.

Lin and Hughson [6] recently reported an analogy between turbulence and human heart rate dynamics by finding a similarity of the structure function—directly linked with the multifractal formalism [11]—of heart rate increments to that of spatial velocity differences in a random cascade process proposed as a model of hydrodynamic turbulence. One of the common features of such cascade-type multifractal models is the evolution in the shape of the PDF of the increments from Gaussian at large scales to stretched exponential at smaller scales [12]. In this study, we disprove this cascadelike assertion and demonstrate that heart rate signals do not follow the evolution in the shape of the (increment) PDF characteristic for cascadelike processes, but report for the first time a *robust* scale invariance. As long-range correlation, non-Gaussianity, and multifractality are also typical characteristics of a system at the critical point [13,14], and fluctuations in a system at a critical point are generally associated with the scale invariance and universal behavior of the scaling function [15,16], we conclude that such robust scale invariance in the increment PDF suggests the alternative scenario of the near critical statelike operation for the healthy heart rate dynamics.

The long-term heart rate data analyzed have been measured as sequential heart interbeat intervals $b(i)$, where i is the beat number. We investigate the PDF of heart rate increments at different time scales (in beat numbers), where the nonstationarity of the data has been eliminated by local detrending [1]. We first integrate the $b(i)$, $B(m) = \sum_{j=1}^{m} b(j)$, and the resultant $B(m)$ is divided into sliding segments of size $2s$. Then in each segment the best qth order polynomial is fit to the data. The differences $\Delta_s B(i) = B^*(i + s) - B^*(i)$ at a scale s are obtained by sliding in time over the segments, where $B^*(i)$ is a deviation from the polynomial fit. By this procedure, the $(q - 1)$th order polynomial trends are eliminated and we analyze the whole PDF of $\Delta_s B(i)$.

Using this method, we analyze two experimental and two computer-generated data sets. The first data set consists of daytime (12:00–18:00 h) heart rate data with a length of up to 4×10^4 heartbeats from 50 healthy subjects (10 females and 40 males; ages 21–76 years) without any known disease affecting the autonomic control of heart rate [Fig. 1(a)]. Details of the recruitment of the subjects, screening for medical problems, protocols, and the data collection are described in Sakata *et al.* [18]. The data were collected during normal daily life. The second experimental data set consists of data of seven 26-h-long periods (up to 10^5 beats), collected when the subjects

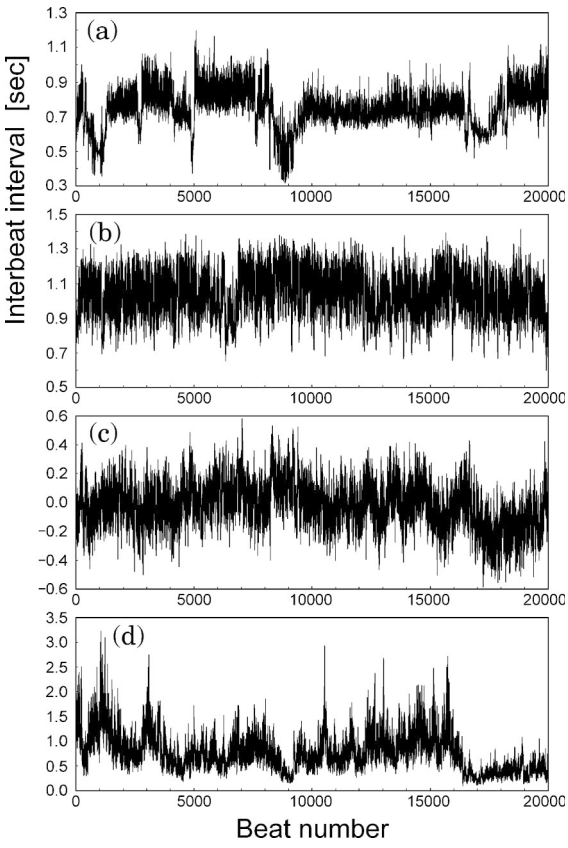

FIG. 1. (a) A representative record of daytime (12:00–18:00 h) heart interbeat intervals for a healthy subject. (b) An example of heartbeat intervals for a healthy subject during constant routine protocol [4,17]. (c) The surrogate data for (b). (d) Data generated by a cascade heart rate increment model [6]. The parameters used for the simulation are $J = 15$, $R_t = 2$, and $\sigma_j = R_t^{-2.5-j/J}$ (see Ref. [6] for details).

(7 males; ages 21–30 years) underwent "constant routine" (CR) protocol, where known behavioral factors affecting heart rate (e.g., exercise, diet, postural changes, and sleep) are eliminated [Fig. 1(b)] [4,17].

In order to test the possible presence of nonlinear mechanisms in complex heart rate dynamics, we apply the surrogate data test to the CR protocol data [19]. We generate a surrogate data set with the same Fourier amplitudes and distributions as the original increments in the CR protocol data [Fig. 1(c)]. Since only linear temporal correlation of $b(i+1) - b(i)$ is retained in the surrogate data, a comparison with the raw data can be used to test whether the PDF of "velocity" increments $\Delta_s B(i)$ possesses some nonlinear mechanism inherent to it. Finally, we generate heart rate increments of comparable lengths, following the "turbulencelike" scenario from the random cascade model proposed recently by Lin and Hughson [Fig. 1(d)] [6].

PDF's of $\Delta_s B(i)$ for healthy humans, which are standardized by dividing the heart rate increments in each record by the standard deviation, are non-Gaussian in shape for a wide range of scales $8 \leq s \leq 4096$ irrespective of whether the subjects were in their normal daily routine [Fig. 2(a)] or in CR [Fig. 2(b)]. In contrast, the PDF's of the surrogate data are near Gaussian, although non-Gaussianity with the fat tails close to those in the observed data is still encountered at fine scales [Fig. 2(c)]. The difference between the healthy human and surrogate data indicates that the observed non-Gaussian behavior is related to nonlinear features of the healthy heart rate dynamics. The PDF's of the cascade model show continuous deformation and the appearance of fat tails when going from large to small scales [Fig. 2(d)].

For a quantitative comparison, we fit the data to the following function based on Castaing's equation [12]:

$$\tilde{P}_s(x) = \int P_L\left(\frac{x}{\sigma}\right)\frac{1}{\sigma} G_{s,L}(\ln\sigma) d(\ln\sigma),$$

where P_L is the increment PDF at a large scale $L > s$, and the self-similarity kernel $G_{s,L}$ determines the nature of the cascade-type multiplicative process. Here we assume P_L and $G_{s,L}$ are both Gaussian,

$$G_{s,L}(\ln\sigma) = \frac{1}{\sqrt{2\pi}\lambda} \exp\left(-\frac{\ln^2\sigma}{2\lambda^2}\right),$$

and investigate the scale dependence of λ^2. The fit of the PDF of actual heart rate increments to Castaing's equation is indeed almost perfect, especially within ± 3 times standard deviation, even for a single record [Fig. 3(a)], and robust in terms of the effect of the order of detrending polynomials on the estimation of λ^2, if the order is greater than 2 [Fig. 3(b)]. In the following, we use the third order detrending for the estimation of λ^2.

Within the turbulent cascade picture, the parameter λ^2 can be interpreted as being proportional to the number of cascade steps and is known to decrease linearly with log$s$ [12,20]. The cascade heart rate model studied here [6] also shows this effect [Fig. 3(c)]. In contrast, the scale dependency of λ^2 for healthy heart rate increments is remarkably different [Fig. 3(c)]. Especially during CR, we cannot see any decrease in λ^2 with log$s$. There is no significant difference in the average λ^2 at different scales, tested by the analysis of variance, over the range of 23–2048 beats for healthy subjects during daily routine and 8–4096 beats during CR [$F(13\,686) = 1.73$ and $F(18\,114) = 1.70$, respectively, $p > 0.05$], and the slopes of λ^2 vs log$s$ are much closer to zero, which means the absence of cascade steps across the scales in the corresponding range.

In addition, when the PDF's at different scales are superimposed [Fig. 3(d)], all the data collapse on the same curve, which is one of the characteristic features observed in fluctuations at a critical point [21]. The range of scales where this scale invariance of PDF is observed, spanning from about ten beats to a few thousand heartbeats, is compatible with that of the robust, behavioral-independent $1/f$ scaling [17] and multifractality [4] of heart rate. The scale invariance in the PDF is also robust

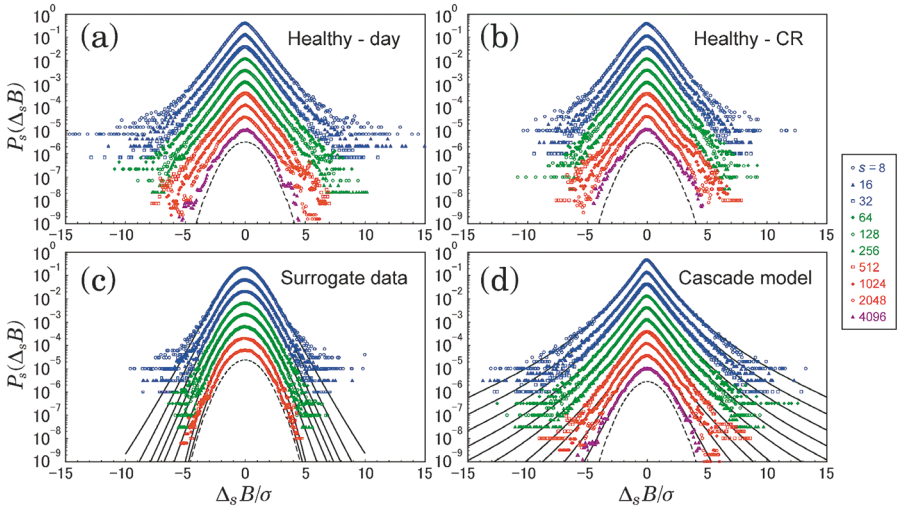

FIG. 2 (color). Deformation of increment PDF's across scales. Standardized PDF's (in logarithmic scale) of $\Delta_s B(i)$ for different time scales are shown for (from top to bottom) $s = 8$, 16, 32, 64, 128, 256, 512, and 1024 beats. These PDF's are estimated from all samples in each group. The dashed line is a Gaussian PDF for comparison. (a) The PDF's from daytime (12:00–18:00 h) heart rate time series from healthy subjects. (b) From healthy subjects during constant routine protocol. (c) From surrogate time series for (b). (d) From a cascade model. Note in the last case (d) the continuous deformation and the appearance of fat tails when going from large to fine scales. In solid lines, we superimpose the deformation of the PDF using Castaing's equation with the log-normal self-similarity kernel, providing an excellent fit to the data.

in the sense that it is observed not only during CR but also during normal daily life, where behavioral modifiers of heart rate dramatically change the mean level of heart rate [e.g., Fig. 1(a)]. We thus find a novel property of scale invariance in healthy human heart rate dynamics, reminiscent of systems in a critical state. In particular, the invariance discovered strongly supports the view that the healthy human heart is controlled to converge continually to a critical state. Such a critical point itself may, however, be shifted by the effects of the external and/or internal environment.

Struzik *et al.* [22] recently demonstrated that modifying the relative importance of either the sympathetic or the parasympathetic branch of the autonomic nervous

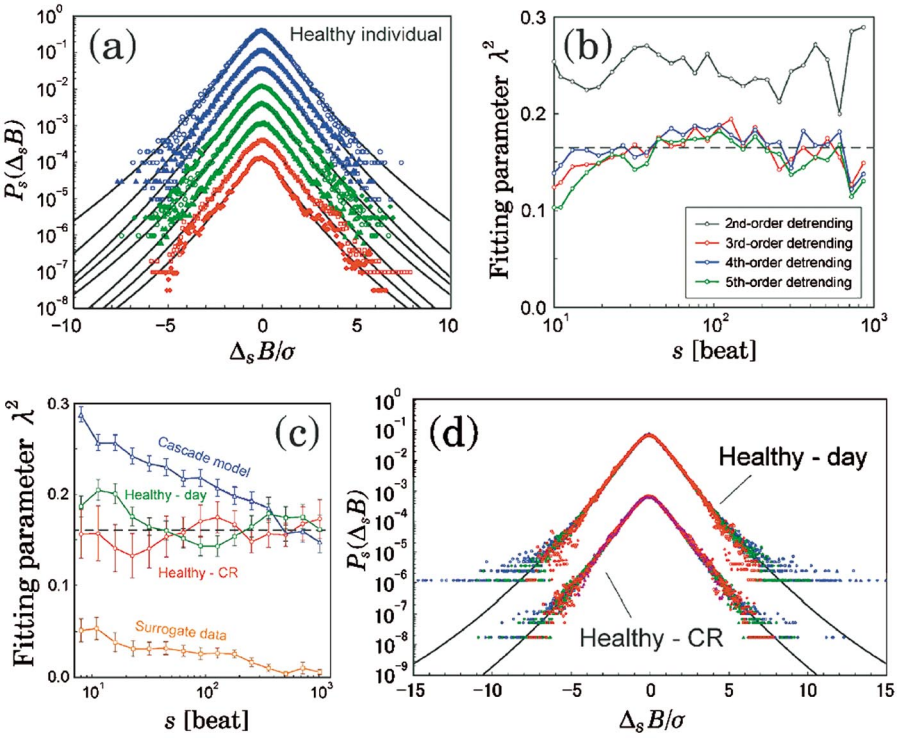

FIG. 3 (color). (a) Standardized PDF for a single subject during constant routine protocol [see Fig. 1(b)]. In solid lines, we superimpose the PDF's using Castaing's equation. (b) Dependence of the fitting parameter λ^2 of Castaing's equation on the order of detrending polynomials. (c) Dependence of the fitting parameter λ^2 on the scale s. The error bars indicate the standard error of the group averages. (d) Superposition of standardized PDF's at different scales shown in Figs. 2(a) and 2(b), where we use the scale range $8 \leq s \leq 2048$ and $8 \leq s \leq 4096$, respectively. In solid lines, we superimpose the PDF using the Castaing's equation ($\lambda^2 = 0.16$ for both groups).

system leads to a substantial decrease in $1/f$ scaling, showing that $1/f$ scaling in healthy heart rate requires the existence of and the intricate balance between the antagonistic activity of these two branches. They further suggest the view of cardiac neuroregulation as a system in a critical state [23], and permanently out of equilibrium, in which concerted interplay of the sympathetic and parasympathetic nervous systems is required for preserving momentary "balance." Our findings provide more direct evidence for this. The precise mechanism responsible for *critical* heart rate dynamics requires further research. It is of note, however, that there exists a physiological model for the dynamics of cardiac neuroregulation [24], equipped with antagonistic and multiplicative delayed feedback loops, within time scales where the critical scale invariance in heart rate is observed. The mechanism of the critical mode of operation could be clarified by investigating essential dynamics in such a "first principles," nonlinear physiological model.

The functional advantage of the heart rate control system being in a critical state remains an open question. However, an analogy with other critical phenomena might help to understand this. Studies on transport properties through complex networks [25,26] have demonstrated maximum efficiency of transportation at the critical point, which is the phase transition point from an "uncrowded" state to a "congested" state in the transportation routes. Thus, our results may indicate that the central neuroregulation continually brings the heart to a critical state to maximize its functional ability, coping with the continually changing preload and afterload on the heart. This may be particularly important in understanding the widely reported evidence that decreased $1/f$ variability [8,9], especially in the low frequency region [8,10], is associated with increased mortality in cardiac patients. To date there have been no successful attempts to provide a satisfactory explanation for this. We suggest for the first time that a breakdown of the optimal control achieved in the critical state might be related to this clinically important phenomenon.

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[1] C. K. Peng *et al.*, Phys. Rev. Lett. **70**, 1343 (1993).
[2] Y. Yamamoto and R. L. Hughson, Am. J. Physiol. **266**, R40 (1994).
[3] P. C. Ivanov *et al.*, Nature (London) **399**, 461 (1999).
[4] L. A. N. Nunes Amaral *et al.*, Phys. Rev. Lett. **86**, 6026 (2001).
[5] U. Frisch, *Turbulence* (Cambridge University Press, Cambridge, 1995), 1st ed.
[6] D. C. Lin and R. L. Hughson, Phys. Rev. Lett. **86**, 1650 (2001).
[7] C. K. Peng *et al.*, Integrative Physiol. Behav. Sci. **29**, 283 (1994).
[8] J. T. Bigger, Jr. *et al.*, Circulation **93**, 2142 (1996).
[9] H. V. Huikuri *et al.*, Circulation **101**, 47 (2000).
[10] J. T. Bigger, Jr. *et al.*, Circulation **85**, 164 (1992).
[11] J. F. Muzy, E. Bacry, and A. Arneodo, Phys. Rev. E **47**, 875 (1993).
[12] B. Castaing, Y. Gagne, and E. J. Hopfinger, Physica (Amsterdam) **46D**, 177 (1990).
[13] H. Hinrichsen, O. Stenull, and H.-K. Janssen, Phys. Rev. E **65**, 045104 (2002).
[14] L. P. Kadanoff *et al.*, Phys. Rev. A **39**, 6524 (1989).
[15] H. E. Stanley, Rev. Mod. Phys. **71**, S358 (1999).
[16] J. P. Sethna, K. A. Dahmen, and C. R. Myers, Nature (London) **410**, 242 (2001).
[17] N. Aoyagi, K. Ohashi, and Y. Yamamoto, Am. J. Physiol. **285**, R171 (2003).
[18] S. Sakata *et al.*, Am. J. Physiol. **276**, R1724 (1999).
[19] T. Schreiber and A. Schmitz, Physica (Amsterdam) **142D**, 346 (2000).
[20] S. Ghashghaie *et al.*, Nature (London) **381**, 767 (1996).
[21] A. P. Mehta *et al.*, Phys. Rev. E **65**, 046139 (2002).
[22] Z. R. Struzik *et al.*, Phys. Rev. E (to be published).
[23] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).
[24] K. Kotani *et al.*, Phys. Rev. E **65**, 051923 (2002).
[25] M. Takayasu, H. Takayasu, and K. Fukuda, Physica (Amsterdam) **277A**, 248 (2000).
[26] S. Valverdea and R. V. Solé, Physica (Amsterdam) **312A**, 636 (2002).