Supplemental Material

Gain-field encoding of the kinematics of both arms in the internal model enables flexible bimanual action

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Theoretical relationship between the generalization function and the primitives

State-space model and trial-dependent adaptation to a constant force field

We assumed that the output force is constructed by a linear summation of the output of primitives as:

$$\hat{f}^{(i)} = \left[\boldsymbol{w}^{(i)}\right]^t \boldsymbol{g}(\theta_r^{(i)}, \theta_l^{(i)}) \tag{S1}$$

where i is the trial number, and $\boldsymbol{g}(\theta_r, \theta_l) = [g_1(\theta_r, \theta_l), g_2(\theta_r, \theta_l), \cdots, g_N(\theta_r, \theta_l)]^t$ and $\boldsymbol{w} = [w_1, w_2, \cdots, w_N]^t$ are column vectors whose elements represent the output and weight of each primitive, respectively.

A state space model of the motor adaptation to the force field f can be represented as:

$$e^{(i)} = d(\theta_l^{(i)})(f^{(i)} - \hat{f}^{(i)}) \tag{S2}$$

$$\boldsymbol{w}^{(i+1)} = \alpha \boldsymbol{w}^{(i)} + e^{(i)} K \boldsymbol{g}(\theta_r^{(i)}, \theta_l^{(i)}) \quad \text{(S3)}$$

where e is the movement error, $d(\theta_l)$ is the compliance that depends on the movement direction of the trained arm (here, we assume that the left arm is trained), and α and K are constants representing, respectively, the spontaneous loss of memory and the update rate to the error.

From these equations, we can obtain the trial-dependent changes in the movement error when a constant force f is imposed for only a particular movement combination (θ_r, θ_l) as:

$$e^{(n)} = \frac{K[d(\theta_l)]^2 f \boldsymbol{g}^t \boldsymbol{g}}{1-\alpha+Kd(\theta_l)\boldsymbol{g}^t \boldsymbol{g}} [\alpha - Kd(\theta_l)\boldsymbol{g}^t \boldsymbol{g}]^{(n-1)} + \frac{(1-\alpha)d(\theta_l)f}{1-\alpha+Kd(\theta_l)\boldsymbol{g}^t \boldsymbol{g}} \quad \text{(S4)}$$

where $\boldsymbol{g}$ is the abbreviation of $\boldsymbol{g}(\theta_r, \theta_l)$.

From Eqs. (S1)-(S3), the weight vector after sufficient training of a constant force field f is obtained as:

$$\boldsymbol{w}^t = \frac{Kd(\theta_l) f \boldsymbol{g}(\theta_r,\theta_l)^t}{1-\alpha+Kd(\theta_l)\boldsymbol{g}(\theta_r,\theta_l)^t \boldsymbol{g}(\theta_r,\theta_l)}. \quad \text{(S5)}$$

Therefore, the output force can be represented as:

$$\hat{f}(\theta_r, \theta_l) = \frac{Kd(\theta_l)\boldsymbol{g}(\theta_r,\theta_l)^t \boldsymbol{g}(\theta_r,\theta_l)}{1-\alpha+Kd(\theta_l)\boldsymbol{g}(\theta_r,\theta_l)^t \boldsymbol{g}(\theta_r,\theta_l)} f. \quad \text{(S6)}$$

When movement directions of both arms are changed by $\Delta\theta_r$ and $\Delta\theta_l$, the force output (i.e., aftereffect) is represented as:

$$\hat{f}(\theta_r + \Delta\theta_r, \theta_l + \Delta\theta_l) = \boldsymbol{w}^t \boldsymbol{g}(\theta_r + \Delta\theta_r, \theta_l + \Delta\theta_l)$$

$$= \frac{Kd(\theta_l)\boldsymbol{g}(\theta_r,\theta_l)^t \boldsymbol{g}(\theta_r+\Delta\theta_r,\theta_l+\Delta\theta_l)}{1-\alpha+Kd(\theta_l)\boldsymbol{g}(\theta_r,\theta_l)^t \boldsymbol{g}(\theta_r,\theta_l)} f. \quad \text{(S7)}$$

Thus, the function of how the training effect is transferred from (θ_r, θ_l) to $(\theta_r + \Delta\theta_r, \theta_l + \Delta\theta_l)$ is represented by:

$$\Phi(\Delta\theta_r, \Delta\theta_l) = \frac{\hat{f}(\theta_r+\Delta\theta_r,\theta_l+\Delta\theta_l)}{\hat{f}(\theta_r,\theta_l)} = \frac{\boldsymbol{g}(\theta_r,\theta_l)^t \boldsymbol{g}(\theta_r+\Delta\theta_r,\theta_l+\Delta\theta_l)}{\boldsymbol{g}(\theta_r,\theta_l)^t \boldsymbol{g}(\theta_r,\theta_l)}. \quad \text{(S8)}$$

Decomposition of the generalization function: Multiplicative encoding

If the primitives encode the movement directions of both hands multiplicatively as: $g_j(\theta_r, \theta_l) = r_j(\theta_r) l_j(\theta_l)$, then

$$\sum_{j=1}^{N} g_j(\theta_r, \theta_l) g_j(\theta_r + \Delta\theta_r, \theta_l + \Delta\theta_l) = \sum_{j=1}^{N} r_j(\theta_r)\, l_j(\theta_l) r_j(\theta_r + \Delta\theta_r) l_j(\theta_r + \Delta\theta_l)$$

$$= \sum_{j=1}^{N} r_j(\theta_r)\, r_j(\theta_r + \Delta\theta_r) l_j(\theta_l) l_j(\theta_l + \Delta\theta_l). \quad \text{(S9)}$$

When N is sufficiently large (N is assumed to be a square number), and $l_j(\theta_l)$ and $r_j(\theta_r)$ have translational symmetry with respect to j and are distributed uniformly on the (θ_r, θ_l) plane, then

$$\sum_{j=1}^{N} g_j(\theta_r, \theta_l) g_j(\theta_r + \Delta\theta_r, \theta_l + \Delta\theta_l)$$

$$\approx \frac{1}{N} \sum_{j=1}^{\sqrt{N}} r_j(\theta_r)\, r_j(\theta_r + \Delta\theta_r) \sum_{j=1}^{\sqrt{N}} l_j(\theta_l) l_j(\theta_l + \Delta\theta_l). \quad \text{(S10)}$$

Thus, the transfer function is:

$$\Phi(\Delta\theta_r, \Delta\theta_l) = \frac{\sum_{j=1}^{N} g_j(\theta_r, \theta_l) g_j(\theta_r + \Delta\theta_r, \theta_l + \Delta\theta_l)}{\sum_{i=1}^{N} g_j(\theta_r, \theta_l) g_j(\theta_r, \theta_l)}$$

$$= \frac{\sum_{j=1}^{\sqrt{N}} r_j(\theta_r)\, r_j(\theta_r + \Delta\theta_r) \sum_{j=1}^{\sqrt{N}} l_j(\theta_l) l_j(\theta_l + \Delta\theta_l)}{\sum_{j=1}^{\sqrt{N}} r_j(\theta_r)\, r_j(\theta_r) \sum_{j=1}^{\sqrt{N}} l_j(\theta_l) l_j(\theta_l)}$$

$$= \Phi(\Delta\theta_r, 0)\Phi(0, \Delta\theta_l). \qquad \text{(S11)}$$

Decomposition of the generalization function: Additive encoding

If the primitives encode the movement directions of both hands additively as: $g_j(\theta_r, \theta_l) = r_j(\theta_r) + l_j(\theta_l)$, then

$$\sum_{j=1}^{N} g_j(\theta_r, \theta_l) g_j(\theta_r + \Delta\theta_r, \theta_l + \Delta\theta_l)$$

$$= \sum_{j=1}^{N} [r_j(\theta_r) + l_j(\theta_l)][r_j(\theta_r + \Delta\theta_r) + l_j(\theta_l + \Delta\theta_l)]$$

$$= \sum_{j=1}^{N} [r_j(\theta_r) + l_j(\theta_l)]\{[r_j(\theta_r + \Delta\theta_r) + l_j(\theta_l)]$$

$$+[r_j(\theta_r) + l_j(\theta_l + \Delta\theta_l)] - [r_j(\theta_r) + l_j(\theta_l)]\}. \qquad \text{(S12)}$$

Thus, the transfer function is:

$$\Phi(\Delta\theta_r, \Delta\theta_l) = \Phi(\Delta\theta_r, 0) + \Phi(0, \Delta\theta_l) - 1. \qquad \text{(S13)}$$

It should be noted that a previous work (Wainscott et al., 2005) has obtained theoretically similar relationships (Eqs.(S11) and (S13)) in the generalization function calculated from the trial-by-trial changes in the aftereffects.

Special case: Gaussian encoding

Here, we assume that the encoding function can be represented by a Gaussian function. In the case of multiplicative and additive encoding, the primitive can be represented, respectively, as:

$$g_j(\theta_r, \theta_l) = \left\{a_r \exp\left[\frac{-({\varphi_r}_j-\theta_r)^2}{2\sigma_r^2}\right] + b_r\right\}\left\{a_l \exp\left[\frac{-({\varphi_l}_j-\theta_l)^2}{2\sigma_l^2}\right] + b_l\right\} \quad \text{(S14)}$$

and

$$g_j(\theta_r, \theta_l) = a_r \exp\left[\frac{-({\varphi_r}_j-\theta_r)^2}{2\sigma_r^2}\right] + a_l \exp\left[\frac{-({\varphi_l}_j-\theta_l)^2}{2\sigma_l^2}\right] + b \quad \text{(S15)}$$

where a and b are constants, and φ indicates the preferred direction.

Multiplicative case:

The numerator of Eq. (S8) can be represented as:

$$\sum_{j=1}^{N} g_j(\theta_r, \theta_l) g_j(\theta_r + \Delta\theta_r, \theta_l + \Delta\theta_l)$$

$$= \sum_{j=1}^{N} \left\{a_r \exp\left[\frac{-({\varphi_r}_j-\theta_r)^2}{2\sigma_r^2}\right] + b_r\right\}\left\{a_l \exp\left[\frac{-({\varphi_l}_j-\theta_l)^2}{2\sigma_l^2}\right] + b_l\right\}$$

$$\left\{a_r \exp\left[\frac{-({\varphi_r}_j-\theta_r-\Delta\theta_r)^2}{2\sigma_r^2}\right] + b_r\right\}\left\{a_l \exp\left[\frac{-({\varphi_l}_j-\theta_l-\Delta\theta_l)^2}{2\sigma_l^2}\right] + b_l\right\}. \quad \text{(S16)}$$

If N is sufficiently large and the preferred directions are uniformly distributed,

$$\approx \sum_{j=1}^{\sqrt{N}} \left\{a_r \exp\left[\frac{-(\varphi_{r_j}-\theta_r)^2}{2\sigma_r^2}\right] + b_r\right\}\left\{a_r \exp\left[\frac{-(\varphi_{r_j}-\theta_r-\Delta\theta_r)^2}{2\sigma_r^2}\right] + b_r\right\}$$

$$\times \sum_{j=1}^{\sqrt{N}} \left\{a_l \exp\left[\frac{-(\varphi_{l_j}-\theta_l)^2}{2\sigma_l^2}\right] + b_l\right\}\left\{a_l \exp\left[\frac{-(\varphi_{l_j}-\theta_l-\Delta\theta_l)^2}{2\sigma_l^2}\right] + b_l\right\}, \quad \text{(S17)}$$

then each summation in Eq. (S17) can be expanded as:

$$\sum_{j=1}^{\sqrt{N}} \left\{a \exp\left[\frac{-(\varphi_j-\theta)^2}{2\sigma^2}\right] + b\right\}\left\{a \exp\left[\frac{-(\varphi_j-\theta-\Delta\theta)^2}{2\sigma^2}\right] + b\right\}$$

$$= a^2 \sum_{j=1}^{\sqrt{N}} \exp\left\{-\frac{(\varphi_j-\theta)^2+(\varphi_j-\theta-\Delta\theta)^2}{2\sigma^2}\right\}$$

$$+ab \sum_{j=1}^{\sqrt{N}} \left[\exp\left\{\frac{-(\varphi_j-\theta)^2}{2\sigma^2}\right\} + \exp\left\{\frac{-(\varphi_j-\theta-\Delta\theta)^2}{2\sigma^2}\right\}\right] + \sqrt{N}b^2. \quad \text{(S18)}$$

The first term of the right side of Eq. (S18) can be rewritten as:

$$\exp\left\{-\frac{(\varphi_j-\theta)^2+(\varphi_j-\theta-\Delta\theta)^2}{2\sigma^2}\right\} = \exp\left\{-\frac{\left(\varphi_j-\frac{2\theta+\Delta\theta}{2}\right)^2}{\sigma^2}\right\}\exp\left\{-\frac{(\Delta\theta)^2}{4\sigma^2}\right\}. \quad \text{(S19)}$$

If N is sufficiently large, the summation of Eq. (S19) can be approximated by the integral of the Gaussian function,

$$\sum_{j=1}^{\sqrt{N}} \exp\left\{-\frac{\left(\varphi_j-\frac{2\theta+\Delta\theta}{2}\right)^2}{\sigma^2}\right\} = \frac{\sqrt{N}}{2\pi}\int \exp\left\{-\frac{\left(\varphi-\frac{2\theta+\Delta\theta}{2}\right)^2}{\sigma^2}\right\}d\varphi = \frac{\sqrt{N}\sigma}{2\sqrt{\pi}}. \quad \text{(S20)}$$

Similarly, the summations in the second term on the right-hand side of Eq. (S18) can be obtained from the following equation:

$$\sum_{j=1}^{\sqrt{N}} \exp\left\{\frac{-(\varphi_j-\theta)^2}{2\sigma^2}\right\} = \sum_{j=1}^{\sqrt{N}} \exp\left\{\frac{-(\varphi_j-\theta-\Delta\theta)^2}{2\sigma^2}\right\}$$

$$= \frac{\sqrt{N}}{2\pi}\int \exp\left\{-\frac{(\varphi-\theta)^2}{2\sigma^2}\right\}d\varphi = \frac{\sqrt{N}\sigma}{\sqrt{2\pi}}. \quad \text{(S21)}$$

Therefore,

$$\sum_{j=1}^{N} g_j(\theta_r, \theta_l) g_j(\theta_r + \Delta\theta_r, \theta_l + \Delta\theta_l)$$

$$= \left\{ {a_r}^2 \frac{\sigma_r}{2\sqrt{\pi}} \exp\left[-\frac{(\Delta\theta_r)^2}{4\sigma_r^2} \right] + 2a_r b_r \frac{\sigma_r}{\sqrt{2\pi}} + b_r^2 \right\} \left\{ {a_l}^2 \frac{\sigma_l}{2\sqrt{\pi}} \exp\left[-\frac{(\Delta\theta_l)^2}{4\sigma_l^2} \right] + 2a_l b_l \frac{\sigma_l}{\sqrt{2\pi}} + b_l^2 \right\},$$

(S22)

And, consequently,

$$\Phi(\Delta\theta_r, \Delta\theta_l) = \frac{\sum_{j=1}^{N} g_j(\theta_r, \theta_l) g_j(\theta_r + \Delta\theta_r, \theta_l + \Delta\theta_l)}{\sum_{j=1}^{N} g_j(\theta_r, \theta_l) g_j(\theta_r, \theta_l)}$$

$$= \frac{\left\{ {a_r}^2 \sigma_r \exp\left[-\frac{(\Delta\theta_r)^2}{4\sigma_r^2} \right] + 2\sqrt{2} a_r b_r \sigma_r + 2\sqrt{\pi} b_r^2 \right\} \left\{ {a_l}^2 \sigma_l \exp\left[-\frac{(\Delta\theta_l)^2}{4\sigma_l^2} \right] + 2\sqrt{2} a_l b_l \sigma_l + 2\sqrt{\pi} b_l^2 \right\}}{\left\{ {a_r}^2 \sigma_r + 2\sqrt{2} a_r b_r \sigma_r + 2\sqrt{\pi} b_r^2 \right\} \left\{ {a_l}^2 \sigma_l + 2\sqrt{2} a_l b_l \sigma_l + 2\sqrt{\pi} b_l^2 \right\}}.$$

(S23)

Additive model case:

Similar to the case of the multiplicative encoding model under the assumption that N is sufficiently large and the preferred directions are uniformly distributed, additive encoding (Eq.(S15)) predicts $\Phi(\Delta\theta_r, \Delta\theta_l)$ as:

$$\Phi(\Delta\theta_r, \Delta\theta_l) =$$

$$\frac{\sqrt{\pi} {a_r}^2 \sigma_r \exp\left[-\frac{(\Delta\theta_r)^2}{4\sigma_r^2} \right] + \sqrt{\pi} {a_l}^2 \sigma_l \exp\left[-\frac{(\Delta\theta_l)^2}{4\sigma_l^2} \right] + 2a_r a_l \sigma_r \sigma_l + 2\sqrt{2\pi} a_r b \sigma_r + 2\sqrt{2\pi} a_l b \sigma_l + 2\pi b^2}{\sqrt{\pi} {a_r}^2 \sigma_r + \sqrt{\pi} {a_l}^2 \sigma_l + 2a_r a_l \sigma_r \sigma_l + 2\sqrt{2\pi} a_r b \sigma_r + 2\sqrt{2\pi} a_l b \sigma_l + 2\pi b^2}.$$

(S24)